



## Teaching Guide

Identifying Data					2019/20
Subject (*)	Ecuacións diferenciais		Code	632G02017	
Study programme	Grao en Tecnoloxía da Enxeñaría Civil				
Descriptors					
Cycle	Period	Year	Type	Credits	
Graduate	Yearly	Second	Basic training	9	
Language	English				
Teaching method	Face-to-face				
Prerequisites					
Department	Matemáticas				
Coordinador	Rodríguez-Vellando Fernández-Carvajal, Pablo	E-mail	pablo.rodriguez-vellando@udc.es		
Lecturers	Colominas Ezponda, Ignasi París López, José Rodríguez-Vellando Fernández-Carvajal, Pablo	E-mail	ignacio.colominas@udc.es jose.paris@udc.es pablo.rodriguez-vellando@udc.es		
Web	caminos.udc.es/info/asignaturas/201				
General description	Resolution of ordinary differential equations				

## Study programme competences

Code	Study programme competences
A1	Capacidad para plantear y resolver los problemas matemáticos que puedan plantearse en el ejercicio de la profesión. En particular, conocer, entender y utilizar la notación matemática, así como los conceptos y técnicas del álgebra y del cálculo infinitesimal, los métodos analíticos que permiten la resolución de ecuaciones diferenciales ordinarias y en derivadas parciales, la geometría diferencial clásica y la teoría de campos, para su aplicación en la resolución de problemas de Ingeniería Civil.
B1	Que los estudiantes hayan demostrado poseer y comprender conocimientos en un área de estudio que parte de la base de la educación secundaria general, y se suele encontrar a un nivel que, si bien se apoya en libros de texto avanzados, incluye también algunos aspectos que implican conocimientos procedentes de la vanguardia de su campo de estudio
B2	Que los estudiantes sepan aplicar sus conocimientos a su trabajo o vocación de una forma profesional y posean las competencias que suelen demostrarse por medio de la elaboración y defensa de argumentos y la resolución de problemas dentro de su área de estudio
B3	Que los estudiantes tengan la capacidad de reunir e interpretar datos relevantes (normalmente dentro de su área de estudio) para emitir juicios que incluyan una reflexión sobre temas relevantes de índole social, científica o ética
B4	Que los estudiantes puedan transmitir información, ideas, problemas y soluciones a un público tanto especializado como no especializado
B5	Que los estudiantes hayan desarrollado aquellas habilidades de aprendizaje necesarias para emprender estudios posteriores con un alto grado de autonomía
B6	Resolver problemas de forma efectiva.
B7	Aplicar un pensamiento crítico, lógico y creativo.
B8	Trabajar de forma colaborativa.
B9	Comportarse con ética y responsabilidad social como ciudadano y como profesional.
B10	Comunicarse de manera efectiva en un entorno de trabajo.
B11	Entender y aplicar el marco legal de la disciplina.
B12	Comprensión de la necesidad de actuar de forma enriquecedora sobre el medio ambiente contribuyendo al desarrollo sostenible.
B13	Comprensión de la necesidad de analizar la historia para entender el presente.
B14	Capacidad para organizar y dirigir equipos de trabajo así como de integrarse en equipos multidisciplinares.
B15	Claridad en la formulación de hipótesis.
B16	Capacidad de autoaprendizaje mediante la inquietud por buscar y adquirir nuevos conocimientos, potenciando el uso de las nuevas tecnologías de la información y así poder enfrentarse adecuadamente a situaciones nuevas.
B17	Capacidad para aumentar la calidad en el diseño gráfico de las presentaciones de trabajos.
B18	Capacidad para aplicar conocimientos básicos en el aprendizaje de conocimientos tecnológicos y en su puesta en práctica.



B19	Capacidad de realizar pruebas, ensayos y experimentos, analizando, sintetizando e interpretando los resultados.
C1	Expresarse correctamente, tanto de forma oral como por escrito, en las lenguas oficiales de la comunidad autónoma.
C2	Dominar la expresión y la comprensión de forma oral e escrita de un idioma extranjero.
C3	Utilizar las herramientas básicas de las tecnologías de la información y las comunicaciones (TIC) necesarias para el ejercicio de su profesión y para el aprendizaje a lo largo de la vida.
C4	Desarrollarse para el ejercicio de una ciudadanía abierta, culta, crítica, comprometida, democrática y solidaria, capaz de analizar la realidad, diagnosticar problemas, formular e implantar soluciones basadas en el conocimiento y orientadas al bien común.
C5	Entender la importancia de la cultura emprendedora y conocer los medios al alcance de las personas emprendedoras.
C6	Valorar críticamente el conocimiento, la tecnología y la información disponible para resolver los problemas con que deben enfrentarse.
C7	Asumir como profesional y ciudadano la importancia del aprendizaje a lo largo de la vida.
C8	Valorar la importancia que tiene la investigación, la innovación y el desarrollo tecnológico en el avance socioeconómico y cultural de la sociedad.

## Learning outcomes

Learning outcomes	Study programme competences		
	A1	B1	C1
Ability to solve mathematical problems that may arise in the exercise of the profession. In particular, know, understand and use mathematical notation and basic concepts that allow solving ordinary differential equations for use in solving problems of Civil Engineering.		B1	C1
		B2	C2
		B3	C3
		B4	C4
		B5	C5
		B6	C6
		B7	C7
		B8	C8
		B9	
		B10	
		B11	
		B12	
		B13	
		B14	
		B15	
		B16	
		B17	
		B18	
		B19	

## Contents

Topic	Sub-topic
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1 First order differential equations

- 1.1. Introduction
  - 1.1.1. Concept of ordinary differential equation, and grades.
  - 1.1.2. Modeling of natural phenomena in terms of mathematical equations. Algebraic, differential and functional equations
  - 1.1.3. Origin of differential calculus: Newton and Leibniz
  - 1.1.4. Examples of Civil Engineering problems that can be written in terms of ODEs: Buckling of pillars, fireplaces oscillatory movement in equilibrium, mixed torsion problem of the catenary, mechanical vibration spring systems, ...
- 1.2. General solutions and particular solutions. Cauchy problem and inverse problem
- 1.3. Integration of differential equations: Analytical methods, graphical and numerical
- 1.4. Existence theorem of uniqueness of solutions of first order ODEs
  - 1.4.1 The method of successive approximations Picard
  - 1.4.2. Picard's theorem for first order differential equations
- 1.5. Differential equations in separate variables
- 1.6. Homogeneous differential equations
  - 1.6.2. Homogeneous functions
  - 1.6.3. Homogeneous solution of differential equations
- 1.7. Reducible to homogeneous differential equations
- 1.8. Exact differential equations
- 1.9. Solving differential equations using integration factors
  - 1.9.2. Factors dependent integration  $x$
  - 1.9.3. Factors dependent integration and
  - 1.9.4. Factors dependent integration
- 1.10. Linear differential equation
- 1.11. Bernoulli differential equation
- 1.12. Riccati differential equation
- 1.13. Application examples: Geometric Problems, flush tanks, dynamic problems, dissolution of substances, thermodynamic problems and persecutions.
- 1.14. Not explicit in the equations derived
  - 1.14.2. Solvable equations
  - 1.14.3. Solvable equations and
  - 1.14.4. Solvable equations  $x$
  - 1.14.5. Lagrange equations
  - 1.14.6. Clairaut equation
- 1.15. Curves and Paths
  - 1.15.2. And isogonal orthogonal to a beam curved trajectories in Cartesian coordinates
  - 1.15.3. Isogonal orthogonal to a beam and curved paths in polar coordinates
  - 1.15.4. Parallel curves to a given curve
  - 1.15.5. Involute curves to a given
  - 1.15.6. Envelope curves to a given family
  - 1.15.7. Geometric problems, some notable planar curves: Lemniscata Bernoulli, cardioid, Hypocycloid, cissoid of Diocles, Pascal snail, Ovals of Cassini
  - 1.15.8. Application to problems related to engineering: flow curves through an embankment dam, parables safety, electrical flow curves between two charges of equal magnitude and opposite sign, ...



2 Second order differential equations

- 2.1. Linear differential equations
  - 2.1.1. Concept. Homogeneous equation and complete equation
  - 2.1.2. Application to solving problems of mathematical physics
  - 2.1.3. Methods of solving linear differential equations
  - 2.1.4. Theorem of existence and uniqueness of linear equations: enunciation
- 2.2. Second order linear equations
  - 2.2.1. Superposition theorem
  - 2.2.2. General solution of the homogeneous linear differential equation of order two
  - 2.2.3. Obtaining the second solution from the first
  - 2.2.4. General solution of the complete equation
  - 2.2.5. Getting the particular solution: Method parameter variation
- 2.3. Linear equations of order n
  - 2.3.1. Superposition theorem
  - 2.3.2. General solution of the linear differential equation of order n homogeneous
  - 2.3.3. General solution of the linear differential equation of order n complete
  - 2.3.4. Homogeneous linear equation with constant coefficients
    - 2.3.4.1. Characteristic equation
    - 2.3.4.2. Real and simple roots
    - 2.3.4.3. And multiple real estate
    - 2.3.4.4. Complex and simple roots
    - 2.3.4.5. Complex and multiple roots
  - 2.3.5. Obtaining particular solutions
    - 2.3.5.1. Method of undetermined coefficients
    - 2.3.5.2. Method of variation of parameters
    - 2.3.5.3. Operational methods of Heaviside
      - 2.3.5.3.1. Overview
      - 2.3.5.3.2. Method of successive integrations
      - 2.3.5.3.3. Decomposition method Simple Fractions
      - 2.3.5.3.4. Method Development Series Polynomial Operators
      - 2.3.5.3.5. Exponential Moving Rule
- 2.4. The Euler-Cauchy
  - 2.4.1. Characteristic equation associated with the Euler-Cauchy
  - 2.4.2. Real and simple roots
  - 2.4.3. And multiple real estate
  - 2.4.4. Complex and simple roots
  - 2.4.5. Complex and multiple roots
- 2.5. Resolution of other equations of order n nonlinear
  - 2.5.1. Second-order equations in which does not appear and
  - 2.5.2. Second-order equations in which there appears x
  - 2.5.3. Equations of order n in which there appear
- 2.6. Troubleshooting Free and forced vibrations with and without damping, resonance and tap: Mechanical Systems of springs, balance swings in fireplaces, Archimedes' principle, pendulums, ...
- 2.7. Application problems: geometric, mechanical, electrical, cinematic, ...
- 2.8. Susceptible civil engineering problems to be solved by integrating a differential equation of order greater than one: heavy Cables, antifunicularidad, bows, ...



3 Resolución de ecuacións diferenciais en MATLAB	<ul style="list-style-type: none"><li>3.1. Introduction to MATLAB<ul style="list-style-type: none"><li>3.1.1 . Basic operations</li><li>3.1.2 . Matrices</li><li>3.1.3 . Graphics</li></ul></li><li>3.2 . MATLAB programming</li><li>3.3 . Solving ODEs<ul style="list-style-type: none"><li>3.3.1 . First order equations</li><li>3.3.2 . Higher-order equations</li><li>3.3.3 . Numerical methods</li><li>3.3.4 . Systems</li><li>3.3.5 . Laplace transform</li><li>3.3.6 . Power Series</li></ul></li></ul>
4 Systems of differential equations	<ul style="list-style-type: none"><li>4.1. Introduction to Differential Equations Systems<ul style="list-style-type: none"><li>4.1.1. System concept of Ordinary Differential Equations. Initial value problems</li><li>4.1.2. Systems of linear equations of order n with m equations and unknowns</li><li>4.1.3. Reduction of order na equation system of n equations and unknowns of the first order</li><li>4.1.4. Reduction of a system of order n and m equations and unknowns, one of the first order with n ? m equations and unknowns</li></ul></li><li>4.2. Obtaining the general solution of a linear system of order one<ul style="list-style-type: none"><li>4.2.1. Superposition theorem homogeneous systems solutions</li><li>4.2.2. General solution of a homogeneous system. Fundamental Matrix Solutions</li><li>4.2.3. General solution of a complete system</li></ul></li><li>4.3. Obtaining the general solution of homogeneous systems of linear differential equations with constant coefficients<ul style="list-style-type: none"><li>4.3.1. Method of Laplace Transform</li><li>4.3.2. Disposal Method</li><li>4.3.3. Euler method or the eigenvalues<ul style="list-style-type: none"><li>4.3.3.1. Introduction</li><li>4.3.3.2. Real simple eigenvalues</li><li>4.3.3.3. Complex and simple eigenvalues</li><li>4.3.3.4. Real and multiple eigenvalues<ul style="list-style-type: none"><li>4.3.3.4.1. Default null</li><li>4.3.3.4.2. Greater than or equal to one defect. Concept of Generalized Eigenvectors</li></ul></li></ul></li></ul></li><li>4.4. Getting the particular solution of differential equations Systems Complete<ul style="list-style-type: none"><li>4.4.1. Method of variation of parameters</li><li>4.4.2. Method of undetermined coefficients</li></ul></li><li>4.5. Systems of differential equations Euler-Cauchy</li><li>4.6. Application problems: Problems deposits, mechanical and electrical problems, geometric problems: epicycloid curves and cycloid hipocicloide</li></ul>



5 Laplace Transformed

- 5.1. Definition of the Laplace Transform and the Gamma Function
  - 5.1.1. Definition of the Laplace Transform
  - 5.1.2. Concept of convergence of the Laplace Transform
  - 5.1.3. Application of the Laplace transform to solving ODEs. Analogy with the resolution of ODEs power series
  - 5.1.4. The Gamma Function
  - 5.1.5. Laplace transform of elementary functions
- 5.2. Existence theorem Laplace Transform. Inverse transform and linearity
  - 5.2.1. Concept of piecewise continuous function and function of exponential order
  - 5.2.2. Existence theorem of the Laplace Transform
  - 5.2.3. Uniqueness theorem of the inverse transform
  - 5.2.4. Linearity theorem of the Laplace Transform
- 5.3. Scaling and translations. Heaviside unit step function and Dirac Delta Function
  - 5.3.1. Scaling in t. Compressions and expansions
  - 5.3.2. Translation along s
  - 5.3.3. Heaviside unit step function. Transformed
  - 5.3.4. Translation along t
  - 5.3.5. Dirac delta function. Transformed
- 5.4. Derivatives and integrals
  - 5.4.1. Transformed by the first derivative and the successive derivatives
  - 5.4.2. Transform an integral
  - 5.4.3. Derived from the transformed
  - 5.4.4. Integration of the transformed
- 5.5. Transform of a periodic function
- 5.6. Convolution product
  - 5.6.1. Product definition convolution of two functions
  - 5.6.2. Convolution product properties
- 5.7. Application of the Laplace Transform to the integration of ODEs
  - 5.7.1. Initial value problems. Equations and systems
  - 5.7.2. Getting inverse transforms by partial fractions and convolution product
  - 5.7.3. Application to solving physical problems with step functions and impulse functions, electrical and mechanical problems, ...



6 Resolution of differential equations in power series

- 6.1. Introduction
  - 6.1.1. Justification for the use of power series in solving ODEs
  - 6.1.2. Convergence of power series
  - 6.1.3. Radius of convergence
  - 6.1.4. Analytic functions
- 6.2. Power series solution of first-order ODE
  - 6.2.1. The principle of identity: enunciation
  - 6.2.2. Procedure for obtaining power series solutions to equations of the first order
- 6.3. Solution in powers of second order ODE
  - 6.3.1. Regular and singular points
  - 6.3.2. Existence theorem for power series solutions about ordinary points: enunciation
  - 6.3.3. Procedure for obtaining power series solutions about ordinary points
  - 6.3.4. Legendre differential equation
    - 6.3.4.1. Obtaining the solution of the equation in powers Legendre
    - 6.3.4.2. Legendre polynomials
    - 6.3.4.3. Rodrigues formula
  - 6.3.5. Regular singular points
  - 6.3.6. Existence theorem of Frobenius series solutions: enunciation
  - 6.3.7. Obtaining solutions of ODEs power series about regular singular point: Frobenius method
  - 6.3.8. Bessel differential equation
    - 6.3.8.1. Bessel differential equation  $a$  &  $b$ ; # 61550;
    - 6.3.8.2. Resolution Bessel differential equation in powers
    - 6.3.8.3. Bessel functions of first and second species
    - 6.3.8.4. Bessel's differential equation of order 0
    - 6.3.8.5. Bessel differential equation of the second kind
  - 6.3.9. Resolution power series of equations Chebyshev, Laguerre, Airy, Hermite, hypergeometric Gauss hypergeometric Kummer
  - 6.3.10. Application to the resolution of mechanical, thermal, buckling of pillars problems, ...



<p>7 Resolution of differential equations in series of ortogonal functions. Fourier series. Boundary problems</p>	<p>7.1. Orthogonal functions</p> <p>7.1.1. Concept of orthogonal functions</p> <p>7.1.2. Standard function and orthonormal functions</p> <p>7.1.3. Generalized Fourier series</p> <p>7.1.4. Determination of generalized Fourier coefficients</p> <p>7.1.5. Orthogonal functions with regard to a weighting function</p> <p>7.2. Boundary value problems. The Sturm-Liouville</p> <p>7.2.1. The Sturm-Liouville problem. Eigenvalues ??and eigenfunctions</p> <p>7.2.2. Orthogonality theorem</p> <p>7.2.3. Real character of the eigenvalues</p> <p>7.2.4. Study of the orthogonality of the Hermite polynomials, Laguerre, Legendre and Chevyshev</p> <p>7.2.5. Troubleshooting contour arising in the theory of structural design. Determination of critical loads of Euler</p> <p>7.3. Fourier series</p> <p>7.3.1. Fourier Series concept and application to solving ODEs</p> <p>7.3.2. Fourier series of functions of period and 2L</p> <p>7.3.3. Determining the Fourier coefficients</p> <p>7.3.4. Theorem Convergence of Fourier Series</p> <p>7.3.5. Fourier series of odd and even functions</p> <p>7.3.6. Odd and even non-periodic extensions of functions</p> <p>7.3.7. Complex form of the Fourier series</p> <p>7.3.8. Solving ODEs Fourier series. Resonance</p> <p>7.3.9. Resolution of geometrical, mechanical and electrical differential problems by the Fourier series</p> <p>7.3.10. SF implementation of the resolution of problems related to Civil Engineering plate deformation, joint twisting, warping of sections</p> <p>7.4. Introduction to the Fourier Transform</p> <p>7.4.1. Extension of the concept of Fourier series nonperiodic functions</p> <p>7.4.2. Fourier integral</p> <p>7.4.3. Integral theorem of Fourier. Enunciation</p> <p>7.4.4. Fourier Transform Breast</p> <p>7.4.5. Fourier cosine transform</p> <p>7.4.6. Fourier Transform</p> <p>7.4.6.1. Complex form of the Fourier integral</p> <p>7.4.6.2. Fourier transform</p>
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Planning				
Methodologies / tests	Competencies	Ordinary class hours	Student?s personal work hours	Total hours
Guest lecture / keynote speech	A1 B8 B9 B10 B11 B12 B13 B14 B15 B1 B2 B3 B4 B5 B6 B7 B16 B17 B18 B19 C1 C2 C3 C4 C5 C6 C7 C8	60	60	120





Seminar	A1 B8 B9 B10 B11 B12 B13 B14 B15 B1 B2 B3 B4 B5 B6 B7 B16 B17 B18 B19 C1 C2 C3 C4 C5 C6 C7 C8	90	0	90
Mixed objective/subjective test	A1 B8 B9 B10 B11 B12 B13 B14 B15 B1 B2 B3 B4 B5 B6 B7 B16 B17 B18 B19 C1 C2 C3 C4 C5 C6 C7 C8	0	5	5
Personalized attention		10	0	10

(\*)The information in the planning table is for guidance only and does not take into account the heterogeneity of the students.

Methodologies	
Methodologies	Description
Guest lecture / keynote speech	<p>These classes constitute the main body of teaching practice and be dedicated to both the exposure of theoretical issues strictly related to the subject, and the resolution of exercises and class issues. The timing of the theoretical and practical classes will vary within the teaching schedule based on the requirements of each subject, and will in any case forward students for your convenience.</p> <p>As for the lectures, they will be exposed as clearly and concretely as possible. During his presentation, it will be addressed in particular to the level of knowledge that the student has at the time of exposing the various individual agenda to complete if some aspect which, although not strictly subject of the document may constitute a gap in knowledge of the student body.</p> <p>I consider very important in any of the classes taught, the fact that classes begin and end on time, which helps to strengthen the relationship of respect with the students. We also try as far as possible to expose the issues in a relaxed, friendly tone. In return, it requested by students a positive, caring and active attitude. Pupils regularly insist on the possibility of existence of doubt.</p> <p>All the exhibitions will be held on the board, except in some very specific question, as the explanation of programming codes of some length, in which case the projection of transparencies will be used. During exhibitions on the board will take care of clarity and size of writing, and colored chalk, especially when graphics are reproduced be used.</p>
Seminar	Seminars are meant to be lectures whose aim is to solve problems of the subject. Throughout the development of the course the students will be proposed to solve various problems whose degree of difficulty will be similar to the final assesment. The lecturer will help the students in the resolution of the exercises. The solutions will be collected or, if required, a delivery date will be proposed. The assesment of these problems will help to improve the final grade in the course. The seminars will also include the MATLAB lectures for which attendance and submission of a coursework will be compulsory.
Mixed objective/subjective test	Completion of a written examination with books and notes which will be constituted by a total of five problems.

Personalized attention	
Methodologies	Description
Seminar Mixed objective/subjective test	It will be very convenient to develop tutorials for developing problems and leaves the original problem of implementation so as to achieve proper development in the subject



Assessment			
Methodologies	Competencies	Description	Qualification
Seminar	A1 B8 B9 B10 B11 B12 B13 B14 B15 B1 B2 B3 B4 B5 B6 B7 B16 B17 B18 B19 C1 C2 C3 C4 C5 C6 C7 C8	Problems	10
Mixed objective/subjective test	A1 B8 B9 B10 B11 B12 B13 B14 B15 B1 B2 B3 B4 B5 B6 B7 B16 B17 B18 B19 C1 C2 C3 C4 C5 C6 C7 C8	Written exam	90
Others			

### Assessment comments

### Sources of information

<b>Basic</b>	<ul style="list-style-type: none"> <li>- Edwards C.H., Penney D.E. (1994). Ecuaciones Diferenciales Elementales y Problemas con Condiciones en la Frontera. Prentice Hall Hispanoamericana. Méjico</li> <li>- Kreyszig E. (1993). Advanced Engineering Mathematics . Wiley. Nueva York</li> <li>- Simmons G. F. (1993). Ecuaciones Diferenciales. Con Aplicaciones y Notas Históricas. McGraw-Hill. Madrid</li> <li>- Vellando P. (2002). Colección de problemas resueltos de ecuaciones diferenciales. CopyBelén. Santiago</li> <li>- Vellando P. (2005). Problemas de ecuaciones diferenciales. Aplicaciones a la ingeniería. CopyBelén. Santiago</li> <li>- Zill D.G. (2002). Ecuaciones Diferenciales con Aplicaciones de Modelado. International Thomson Editores. Méjico</li> <li>- Puig Adam P. (1980). Ecuaciones diferenciales . Nuevas Gráficas</li> </ul>
<b>Complementary</b>	

### Recommendations

#### Subjects that it is recommended to have taken before

Cálculo infinitesimal I/632G02001  
Cálculo infinitesimal II/632G02002  
Física aplicada I/632G02004  
Física aplicada II/632G02005  
Álgebra lineal I/632G02007  
Álgebra lineal II/632G02008

#### Subjects that are recommended to be taken simultaneously

#### Subjects that continue the syllabus

#### Other comments

(\*)The teaching guide is the document in which the URV publishes the information about all its courses. It is a public document and cannot be modified. Only in exceptional cases can it be revised by the competent agent or duly revised so that it is in line with current legislation.